Lösen linearer Gleichungssysteme

Allgemein:

$$A \cdot x = b$$

I: $$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1$$

II: $$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2$$

III: $$a_{31}x_1 + a_{31}x_2 + a_{33}x_3 = b_3$$

II - I: $$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2$$

$$-a_{21}x_1 - \frac{a_{12}a_{21}}{a_{11}}x_2 - \frac{a_{21}a_{13}}{a_{11}}x_3 = -b_1\frac{a_{21}}{a_{11}}$$

$$\underbrace{\left(a_{22} - \frac{a_{12}a_{21}}{a_{11}}\right)}_{\alpha_1}x_2 + \underbrace{\left(a_{23} - \frac{a_{21}a_{13}}{a_{11}}\right)}_{\beta_1}x_3 = \underbrace{b_2 - b_1\frac{a_{21}}{a_{11}}}_{\gamma_1}$$

$$\underbrace{\left(a_{32} - \frac{a_{12}a_{31}}{a_{11}}\right)}_{\alpha_2}x_2 + \underbrace{\left(a_{33} - \frac{a_{31}a_{13}}{a_{11}}\right)}_{\beta_2}x_3 = \underbrace{b_3 - b_1\frac{a_{31}}{a_{11}}}_{\gamma_2}$$

I: $$\alpha_1 x_2 + \beta_1 x_3 = \gamma_1$$

II: $$\alpha_2 x_2 + \beta_2 x_3 = \gamma_2$$

II - I: $$\alpha_2 x_2 + \beta_2 x_3 = \gamma_2$$

$$-\alpha_2 x_2 - \beta_1\frac{\alpha_2}{\alpha_1}x_3 = -\frac{\alpha_2}{\alpha_1}\gamma_1$$

$$\Rightarrow\ x_3\left(\beta_2 - \beta_1\frac{\alpha_2}{\alpha_1}\right) = \gamma_2 - \frac{\alpha_2}{\alpha_1}\gamma_1$$

beliebiges Gleichungssystem
ohne Strategie, nur mittels Gauß-Allgorithmus

$$0{,}043\,x_1 - 4{,}321\,x_2 + 2{,}748\,x_3 = 1{,}361$$

$$-1{,}481\,x_1 + 7{,}328\,x_2 - 0{,}548\,x_3 = -2{,}794$$
$$6{,}318\,x_1 - 1{,}743\,x_2 + 4{,}326\,x_3 = 0{,}491$$

$\alpha_1 = -141{,}495$	$\beta_1 = 4{,}098$
$\alpha_2 = 633{,}143$	$\beta_2 = -399{,}438$
$\gamma_1 = 44{,}081$	$\gamma_2 = -199{,}481$

$$x_3 = \frac{\gamma_2 - \frac{\alpha_2}{\alpha_1}\gamma_1}{\beta_2 - \beta_1 \frac{\alpha_2}{\alpha_1}} = \quad \textbf{– 0,103} \quad \text{gerundet}$$

$$x_2 = (\gamma_2 - \beta_2 x_3)\frac{1}{\alpha_2} = \quad \textbf{– 0,380} \quad \text{gerundet}$$

$$x_1 = \frac{1}{a_{11}}(-a_{12}x_2 - a_{13}x_3 + b_1) = \quad \textbf{0,048} \quad \text{nicht gerundet}$$

mit Pivot-Strategie, ohne Skalierung

$$6{,}318\,x_1 - 1{,}743\,x_2 + 4{,}326\,x_3 = 0{,}491$$
$$0{,}043\,x_1 - 4{,}321\,x_2 + 2{,}748\,x_3 = 1{,}361$$
$$-1{,}481\,x_1 + 7{,}328\,x_2 - 0{,}548\,x_3 = -2{,}794$$

$\alpha_1 = -4{,}309$	$\beta_1 = 2{,}719$
$\alpha_2 = 6{,}919$	$\beta_2 = 0{,}466$
$\gamma_1 = 1{,}358$	$\gamma_2 = -2{,}679$

I: $\alpha_1 x_2 + \beta_1 x_3 = \gamma_1$

II: $\alpha_2 x_2 + \beta_2 x_3 = \gamma_2$

(umdrehen:)

$$6{,}919\,x_2 + 0{,}466\,x_3 = -2{,}679$$
$$-4{,}309\,x_2 + 2{,}719\,x_3 = 1{,}358$$

$$x_3 = \frac{1{,}358 - \frac{-4{,}309}{6{,}919} \cdot (-2{,}679)}{2{,}719 - \frac{-4{,}309}{6{,}919} 0{,}466}$$

$x_3 =$ **– 0,103**

$x_2 =$ **– 0,380**

$x_1 =$ **0,0434**

neues System:

$$0{,}041 x_1 + 27{,}324 x_2 - 112{,}011 x_3 = 43{,}12$$
$$-33{,}492 x_1 + 0{,}437 x_2 - 327{,}17 x_3 = -2{,}471$$
$$127{,}915 x_1 - 13{,}393 x_2 + 0{,}012 x_3 = 118{,}438$$

ohne Pivot, ohne Skalierung, nur Gauß

$\alpha_1 =$ 22320,813	$\beta_1 =$ –91826,444
$\alpha_2 =$ –85247561,2	$\beta_2 =$ 349460,672
$\gamma_1 =$ 35221,311	$\gamma_2 =$ –134410,704

$x_3 = -\frac{127{,}5011771}{1297{,}565351} \approx$ **– 0,098**

$x_2 \approx$ **– 1,175**

$x_1 \approx$ **0,908**

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mit Pivot und Sklalierung

1. und 3. Zeile mit 3 Multiplizieren und
3. Zeile mit 1. Zeile vertauschen:

$$383{,}745 x_1 - 40{,}179 x_2 + 0{,}036 x_3 = 355{,}314$$
$$-33{,}492 x_1 + 0{,}437 x_2 - 327{,}17 x_3 = -2{,}471$$
$$0{,}123 x_1 + 81{,}972 x_2 - 336{,}033 x_3 = 129{,}36$$

$\alpha_1 =$ – 3,070	$\beta_1 =$ – 327,114
$\alpha_2 =$ – 81,985	$\beta_2 =$ – 336,033
$\gamma_1 =$ 28,540	$\gamma_2 =$ 129,246

$$x_3 = -\frac{127{,}5011771}{1297{,}565351} \approx \mathbf{-0{,}098}$$

$$x_2 \approx \mathbf{1{,}175}$$

$$x_1 \approx \mathbf{0{,}908}$$

$$\begin{aligned} -3{,}07\,x_2 - 327{,}114\,x_3 &= 28{,}54 \\ 81{,}985\,x_2 - 336{,}033\,x_3 &= 129{,}246 \end{aligned}$$

$$x_3 \approx \mathbf{-0{,}098}$$

$$x_2 \approx \mathbf{1{,}175}$$

$$x_1 \approx \mathbf{1{,}049}$$